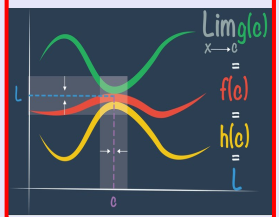
Calculus I

Lecture 22



Feb 19-8:47 AM

More on derivatives

$y = f(x)$

First derivative

y' Y-Prime

$f'(x)$ f - Prime of x

$\frac{dy}{dx}$, $\frac{d}{dx}[f(x)]$

Some rules

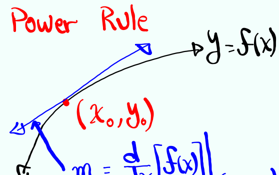
1) $\frac{d}{dx}[c] = 0$

2) $\frac{d}{dx}[x] = 1$

3) $\frac{d}{dx}[x^n] = n\,x^{n-1}$ Power Rule

4) $\frac{d}{dx}[\sin x] = \cos x$

5) $\frac{d}{dx}[\cos x] = -\sin x$



$m = \frac{d}{dx}[f(x)]\Big|_{(x_0, y_0)}$

Oct 7-7:27 AM

New Rules

1) $\frac{d}{dx}\left[f(x) \pm g(x)\right] = \frac{d}{dx}\left[f(x)\right] \pm \frac{d}{dx}\left[g(x)\right]$

2) $\frac{d}{dx}\left[c\,f(x)\right] = c\,\frac{d}{dx}\left[f(x)\right]$

ex: $f(x) = 4\sin x - \frac{1}{2}x^2$

$$f'(x) = \frac{d}{dx}\left[4\sin x\right] - \frac{d}{dx}\left[\tfrac{1}{2}x^2\right]$$

$$= 4\,\frac{d}{dx}\left[\sin x\right] - \frac{1}{2}\,\frac{d}{dx}\left[x^2\right]$$

$$= 4\cdot\cos x - \frac{1}{2}\cdot 2x$$

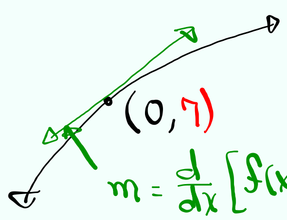
$$= \boxed{4\cos x - x}$$

Oct 7-7:32 AM

Find the eqn of the tan. line to the graph of $f(x) = 3\cos x + x^3 + 4$ at $x = 0$.

$f(0) = 3\cancel{\cos 0}^{\,1} + \cancel{0^3}^{\,0} + 4$

$= 7$



$m = \frac{d}{dx}\left[f(x)\right]\Big|_{(0,7)}$

$f(x) = 3\cos x + x^3 + 4$

$f'(x) = \frac{d}{dx}\left[3\cos x\right] + \frac{d}{dx}\left[x^3\right] + \frac{d}{dx}\left[4\right]$

$= 3\cdot -\sin x + 3x^2 + 0$

$= \boxed{-3\sin x + 3x^2}$

$m = -3\cancel{\sin 0}^{\,0} + 3(0)^2$

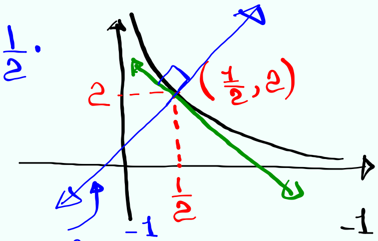
$= 0$

$y - 7 = 0(x - 0)$

$\boxed{y = 7}$

Oct 7-7:37 AM

Find eqn of the normal line to the graph of $f(x) = \frac{1}{x}$ at $x = \frac{1}{2}$.



$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -1 \cdot x^{-1-1} = \frac{-1}{x^2}$

$f'(\frac{1}{2}) = \frac{-1}{(\frac{1}{2})^2} = -4$

$m = \frac{-1}{\frac{d}{dx}[f(x)]\Big|_{(\frac{1}{2},2)}} = \frac{-1}{-4} = \boxed{\frac{1}{4}}$

$y - 2 = \frac{1}{4}(x - \frac{1}{2})$

$y - 2 = \frac{1}{4}x - \frac{1}{8}$

$y = \frac{1}{4}x - \frac{1}{8} + 2$

$y = \frac{1}{4}x - \frac{1}{8} + \frac{16}{8}$ $\boxed{y = \frac{1}{4}x + \frac{15}{8}}$

Oct 7-7:43 AM

More rules

$$\frac{d}{dx}[f(x) \cdot g(x)] = \frac{d}{dx}[f(x)] \cdot g(x) + f(x) \cdot \frac{d}{dx}[g(x)]$$

Product Rule

$$= f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

ex: $\frac{d}{dx}[\sin x \cdot (x^2 + 2x)] = \cos x \cdot (x^2 + 2x) + \sin x \cdot (2x + 2)$

Find $f'(0)$ if $f(x) = (x^3 - 4x) \cdot \cos x$

$f'(x) = (3x^2 - 4) \cdot \cos x + (x^3 - 4x) \cdot -\sin x$

$f'(0) = (3 \cdot 0^2 - 4) \cdot \cos 0 - (0^3 - 4 \cdot 0) \cdot \sin 0 = \boxed{-4}$

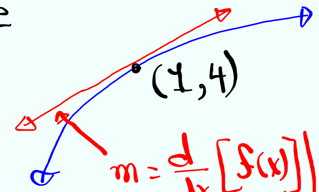
Oct 7-7:49 AM

Find slope of the tan. line to the graph of $f(x) = \sqrt{x}\,(x^2 + 2x + 1)$ at $x = 1$.

$x^{1/2}$

$f(1) = \sqrt{1}(1^2 + 2\cdot 1 + 1)$

$= 4$



$m = \frac{d}{dx}[f(x)]\Big|_{(1,4)} = \boxed{6}$

$f(x) = x^{1/2}(x^2 + 2x + 1)$

$f'(x) = \frac{1}{2}\cdot x^{\frac{1}{2}-1}(x^2 + 2x + 1) + x^{1/2}(2x + 2 + 0)$

$f'(1) = \frac{1}{2}(1)^{-1/2}(1^2 + 2\cdot 1 + 1) + 1^{1/2}(2\cdot 1 + 2)$

$= 2 + 4 = \boxed{6}$

Oct 7-7:58 AM

$f(x) = \sin 2x$ Find $f'(x)$

$\frac{d}{dx}[\sin 2x] = \frac{d}{dx}[2\sin x\cos x]$

2 is the issue.

Chain Rule

$= 2\frac{d}{dx}[\sin x\cos x]$

$= 2[\cos x \cdot \cos x + \sin x \cdot -\sin x]$

$= 2(\underbrace{\cos^2 x - \sin^2 x}) = \boxed{2\cos 2x}$

Oct 7-8:05 AM

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{[g(x)]^2}$$

$$\frac{d}{dx}[\tan x] = \frac{d}{dx}\left[\frac{\sin x}{\cos x}\right] = \frac{\cos x\cdot\cos x - \sin x\cdot{}^{-}\sin x}{[\cos x]^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \boxed{\sec^2 x}$$

$$\frac{d}{dx}[\sin x] = \cos x \qquad \frac{d}{dx}[\cos x] = -\sin x$$

$$\frac{d}{dx}[\tan x] = \sec^2 x$$

Oct 7-8:10 AM

Find $\frac{d}{dx}\left[\frac{2x}{x+2}\right] = \frac{2(x+2) - 2x\cdot 1}{(x+2)^2}$

$$= \boxed{\frac{4}{(x+2)^2}}$$

$$f(x) = \frac{x^2}{x-2}$$

1) $f(1) = \frac{1^2}{1-2} = \frac{1}{-1} = \boxed{-1}$

2) $f'(x) = \frac{2x(x-2) - x^2\cdot 1}{(x-2)^2} = \frac{2x^2 - 4x - x^2}{(x-2)^2} = \boxed{\frac{x^2-4x}{(x-2)^2}}$

3) $f'(1) = \frac{1^2 - 4(1)}{(1-2)^2} = \frac{-3}{1} = \boxed{-3}$

Oct 7-8:16 AM

find eqn of tan. line to the graph of

$f(x) = \dfrac{1}{x^2+1}$ at $x = 1$.

$y - \frac{1}{2} = \frac{-1}{2}(x-1)$

$\boxed{y = \frac{-1}{2}x + 1}$

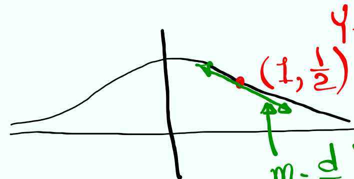
Domain $(-\infty, \infty)$

$f(-x) = \dfrac{1}{(-x)^2+1} = \dfrac{1}{x^2+1} = f(x)$

even function

Symmetric w/t y-axis.

$\lim_{x \to \infty} f(x) = 0$



$(1, \frac{1}{2})$

$m = \frac{d}{dx}\left[f(x)\right]\Big|_{(1,\frac{1}{2})}$

$f'(x) = \dfrac{\underbrace{0 \cdot (x^2+1)}_{\to 0} - 1 \cdot 2x}{(x^2+1)^2}$

$f(x) = \dfrac{1}{x^2+1}$

$f'(x) = \dfrac{-2x}{(x^2+1)^2}$ $\quad m = f'(1) = \dfrac{-2 \cdot 1}{(1^2+1)^2} = \dfrac{-2}{4} = \boxed{\dfrac{-1}{2}}$